Introduction to Information Retrieval

http://informationretrieval.org

IIR 21: Link Analysis

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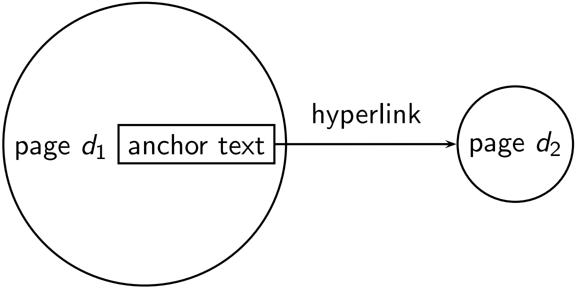
Overview

Outline

The web as a directed graph

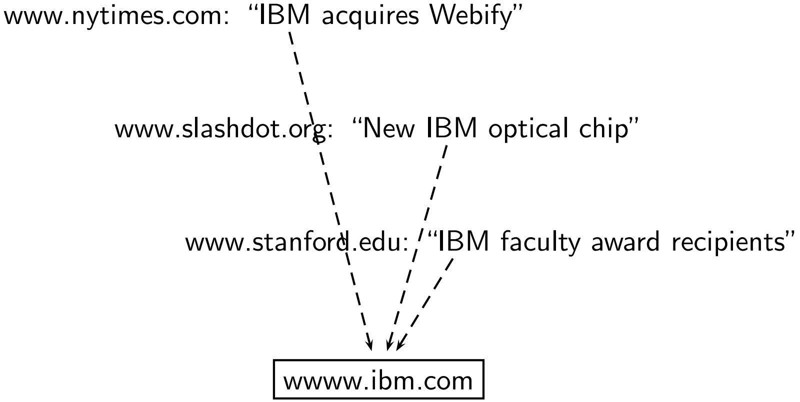


- Assumption 1: A hyperlink is a quality signal.
  - The hyperlink $d_1 \rightarrow d_2$ indicates that d_1's author deems d_2 high-quality and relevant.
- Assumption 2: The anchor text describes the content of d_2.
  - We use anchor text somewhat loosely here for: the text surrounding the hyperlink.
  - Example: "You can find cheap cars here."
  - Anchor text: "You can find cheap cars here"

[text of d_2] only vs. [text of d_2] + [anchor text $\rightarrow d_2$]

- Searching on [text of d_2] + [anchor text $\rightarrow d_2$] is often more effective than searching on [text of d_2] only.
- Example: Query *IBM*
  - Matches IBM's copyright page
  - Matches many spam pages
  - Matches IBM wikipedia article
  - May not match IBM home page!
  - ... if IBM home page is mostly graphics
- Searching on [anchor text $\rightarrow d_2$] is better for the query *IBM*.
  - In this representation, the page with the most occurrences of *IBM* is www.ibm.com. □

Anchor text containing *IBM* pointing to www.ibm.com



Indexing anchor text

- Thus: Anchor text is often a better description of a page's content than the page itself.
- Anchor text can be weighted more highly than document text. (based on Assumptions 1&2) □

Exercise: Assumptions underlying PageRank

- Assumption 1: A link on the web is a quality signal – the author of the link thinks that the linked-to page is high-quality.
- Assumption 2: The anchor text describes the content of the linked-to page.
- Is assumption 1 true in general?
- Is assumption 2 true in general?

Google bombs

- A Google bomb is a search with “bad” results due to maliciously manipulated anchor text.
- Google introduced a new weighting function in 2007 that fixed many Google bombs.
- Still some remnants: [dangerous cult] on Google, Bing, Yahoo
    - Coordinated link creation by those who dislike the Church of Scientology
- Defused Google bombs: [dumb motherf....], [who is a failure?], [evil empire]

Outline

Origins of PageRank: Citation analysis (1)

- Citation analysis: analysis of citations in the scientific literature
- Example citation: "Miller (2001) has shown that physical activity alters the metabolism of estrogens."
- We can view "Miller (2001)" as a hyperlink linking two scientific articles.
- One application of these "hyperlinks" in the scientific literature:
  - Measure the similarity of two articles by the overlap of other articles citing them.
  - This is called cocitation similarity.
  - Cocitation similarity on the web: Google's "related:" operator, e.g. [related:www.ford.com]

Origins of PageRank: Citation analysis (2)

- Another application: Citation frequency can be used to measure the impact of a scientific article.
  - Simplest measure: Each citation gets one vote.
  - On the web: citation frequency = inlink count
- However: A high inlink count does not necessarily mean high quality ...
- ... mainly because of link spam.
- Better measure: weighted citation frequency or citation rank
  - An citation's vote is weighted according to its citation impact.
  - Circular? No: can be formalized in a well-defined way. □

Origins of PageRank: Citation analysis (3)

- Better measure: weighted citation frequency or citation rank
- This is basically PageRank.
- PageRank was invented in the context of citation analysis by Pinsker and Narin in the 1960s.
- Citation analysis is a big deal: The budget and salary of this lecturer are / will be determined by the impact of his publications!

Origins of PageRank: Summary

- We can use the same formal representation for
  - citations in the scientific literature
  - hyperlinks on the web
- Appropriately weighted citation frequency is an excellent measure of quality . . .
  - . . . both for web pages and for scientific publications.
- Next: PageRank algorithm for computing weighted citation frequency on the web □

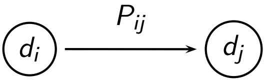
Outline

Model behind PageRank: Random walk

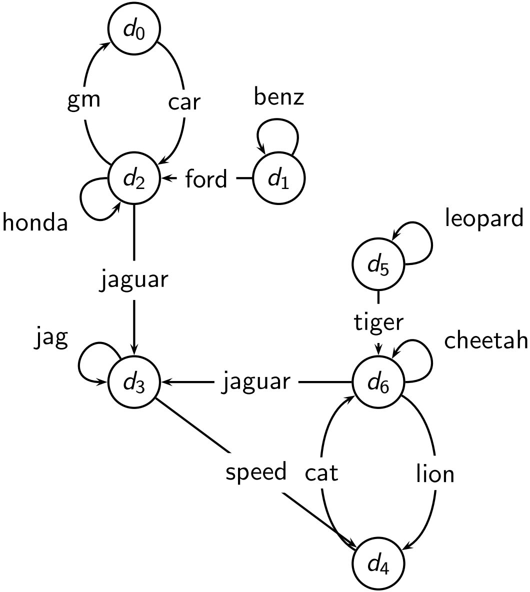
- Imagine a web surfer doing a random walk on the web
  - Start at a random page
  - At each step, go out of the current page along one of the links on that page, equiprobably
- In the steady state, each page has a long-term visit rate.
- This long-term visit rate is the page's PageRank.
- PageRank = long-term visit rate = steady state probability □

Formalization of random walk: Markov chains

- A Markov chain consists of N states, plus an $N \times N$ transition probability matrix P.
- state = page
- At each step, we are on exactly one of the pages.
- For $1 \leq i, j \leq N$, the matrix entry P_{ij} tells us the probability of j being the next page, given we are currently on page i.
- Clearly, for all i, $\sum_{j=1}^{N} P_{ij} = 1$



Example web graph



	PageRank
d_0	0.05
d_1	0.04
d_2	0.11
d_3	0.25
d_4	0.21
d_5	0.04
d_6	0.31

PageRank(d2)< PageRank(d6): why?

	a	h
d_0	0.10	0.03
d_1	0.01	0.04
d_2	0.12	0.33
d_3	0.47	0.18

Link matrix for example

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0	0	1	0	0	0	0
d_1	0	1	1	0	0	0	0
d_2	1	0	1	1	0	0	0
d_3	0	0	0	1	1	0	0
d_4	0	0	0	0	0	0	1
d_5	0	0	0	0	0	1	1
d_6	0	0	0	1	1	0	1

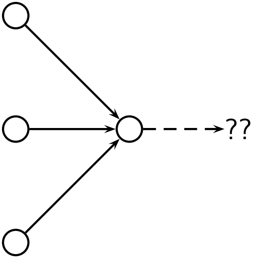
Transition probability matrix P for example

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.00	0.00	1.00	0.00	0.00	0.00	0.00
d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

Long-term visit rate

- Recall: PageRank = long-term visit rate
- Long-term visit rate of page d is the probability that a web surfer is at page d at a given point in time.
- Next: what properties must hold of the web graph for the long-term visit rate to be well defined?
- The web graph must correspond to an ergodic Markov chain.
- First a special case: The web graph must not contain dead ends. □

Dead ends



- The web is full of dead ends.
- Random walk can get stuck in dead ends.
- If there are dead ends, long-term visit rates are not well-defined (or non-sensical).

Teleporting – to get us out of dead ends

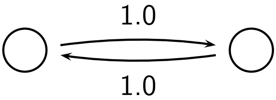
- At a dead end, jump to a random web page with prob. $1/N$.
- At a non-dead end, with probability 10%, jump to a random web page (to each with a probability of $0.1/N$).
- With remaining probability (90%), go out on a random hyperlink.
  - For example, if the page has 4 outgoing links: randomly choose one with probability (1-0.10)/4=0.225
- 10% is a parameter, the teleportation rate.
- Note: "jumping" from dead end is independent of teleportation rate.

Result of teleporting

- With teleporting, we cannot get stuck in a dead end.
- But even without dead ends, a graph may not have well-defined long-term visit rates.
- More generally, we require that the Markov chain be ergodic.

Ergodic Markov chains

- A Markov chain is ergodic iff it is irreducible and aperiodic.
- Irreducibility. Roughly: there is a path from any page to any other page.
- Aperiodicity. Roughly: The pages cannot be partitioned such that the random walker visits the partitions sequentially.



- A non-ergodic Markov chain:

Ergodic Markov chains

- Theorem: For any ergodic Markov chain, there is a unique long-term visit rate for each state.
- This is the steady-state probability distribution.
- Over a long time period, we visit each state in proportion to this rate.
- It doesn't matter where we start.
- Teleporting makes the web graph ergodic.
- $\Rightarrow$ Web-graph+teleporting has a steady-state probability distribution.
- $\Rightarrow$ Each page in the web-graph+teleporting has a PageRank. $\square$

Where we are

- We now know what to do to make sure we have a well-defined PageRank for each page.
- Next: how to compute PageRank

Formalization of “visit”: Probability vector

- A probability (row) vector $\vec{x} = (x_1, \dots, x_N)$ tells us where the random walk is at any point.
- Example:

(	0	0	0	...	1	...	0	0	0	)
	1	2	3	...	i	...	N-2	N-1	N	

- More generally: the random walk is on page i with probability x_i.
- Example:

(	0.05	0.01	0.0	...	0.2	...	0.01	0.05	0.03	)
	1	2	3	...	i	...	N-2	N-1	N	

- $\sum x_i = 1$ □

Change in probability vector

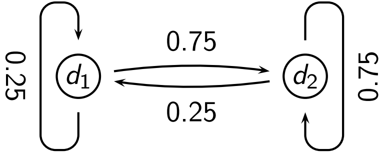
- If the probability vector is $\vec{x} = (x_1, \ldots, x_N)$ at this step, what is it at the next step?
- Recall that row i of the transition probability matrix P tells us where we go next from state i.
- So from $\vec{x}$, our next state is distributed as $\vec{x}P$. □

Steady state in vector notation

- The steady state in vector notation is simply a vector $\vec{\pi} = (\pi_1, \pi_2, \ldots, \pi_N)$ of probabilities.
- (We use $\vec{\pi}$ to distinguish it from the notation for the probability vector $\vec{x}$.)
- π_i is the long-term visit rate (or PageRank) of page i.
- So we can think of PageRank as a very long vector – one entry per page.

Steady-state distribution: Example

- What is the PageRank / steady state in this example?



Steady-state distribution: Example

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$			
			$P_{11} = 0.25$	$P_{12} = 0.75$	PageRank
			$P_{21} = 0.25$	$P_{22} = 0.75$	
t_0	0.25	0.75	0.25	0.75	
t_1	0.25	0.75	(convergence)		

vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$

$$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$$

□

How do we compute the steady state vector?

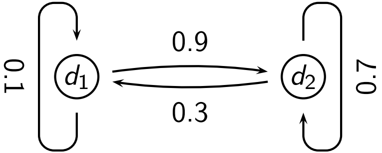
- In other words: how do we compute PageRank?
- Recall: $\vec{\pi} = (\pi_1, \pi_2, \ldots, \pi_N)$ is the PageRank vector, the vector of steady-state probabilities ...
- ... and if the distribution in this step is $\vec{x}$, then the distribution in the next step is $\vec{x}P$.
- But $\vec{\pi}$ is the steady state!
- So: $\vec{\pi} = \vec{\pi}P$
- Solving this matrix equation gives us $\vec{\pi}$.
- $\vec{\pi}$ is the principal left eigenvector for P ...
- ... that is, $\vec{\pi}$ is the left eigenvector with the largest eigenvalue.
- All transition probability matrices have largest eigenvalue 1. □

One way of computing the PageRank $\vec{\pi}$

- Start with any distribution $\vec{x}$, e.g., uniform distribution
- After one step, we're at $\vec{x}P$.
- After two steps, we're at $\vec{x}P^2$.
- After k steps, we're at $\vec{x}P^k$.
- Algorithm: multiply $\vec{x}$ by increasing powers of P until convergence.
- This is called the power method.
- Recall: regardless of where we start, we eventually reach the steady state $\vec{\pi}$.
- Thus: we will eventually (in asymptotia) reach the steady state. □

Power method: Example

- What is the PageRank / steady state in this example?



- The steady state distribution (= the PageRanks) in this example are 0.25 for d_1 and 0.75 for d_2. □

Computing PageRank: Power method

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$			
			$P_{11} = 0.1$ $P_{21} = 0.3$	$P_{12} = 0.9$ $P_{22} = 0.7$	
t_0	0	1	0.3	0.7	$= \vec{x}P$
t_1	0.3	0.7	0.24	0.76	$= \vec{x}P^2$
t_2	0.24	0.76	0.252	0.748	$= \vec{x}P^3$
t_3	0.252	0.748	0.2496	0.7504	$= \vec{x}P^4$
			. . .		. . .
t_∞	0.25	0.75	0.25	0.75	$= \vec{x}P^\infty$

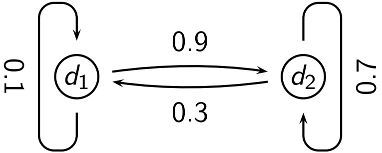
PageRank vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.25, 0.75)$

$$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$$

$$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$$

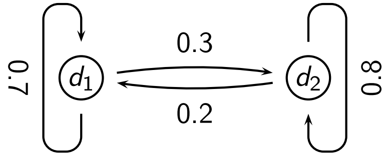
Power method: Example

- What is the PageRank / steady state in this example?



- The steady state distribution (= the PageRanks) in this example are 0.25 for d_1 and 0.75 for d_2. □

Exercise: Compute PageRank using power method



Solution

	x_1 $P_t(d_1)$	x_2 $P_t(d_2)$		
			$P_{11} = 0.7$ $P_{21} = 0.2$	$P_{12} = 0.3$ $P_{22} = 0.8$
t_0	0	1	0.2	0.8
t_1	0.2	0.8	0.3	0.7
t_2	0.3	0.7	0.35	0.65
t_3	0.35	0.65	0.375	0.625
				$\dots$
t_∞	0.4	0.6	0.4	0.6

PageRank

vector $= \vec{\pi} = (\pi_1, \pi_2) = (0.4, 0.6)$

$P_t(d_1) = P_{t-1}(d_1) * P_{11} + P_{t-1}(d_2) * P_{21}$

$P_t(d_2) = P_{t-1}(d_1) * P_{12} + P_{t-1}(d_2) * P_{22}$

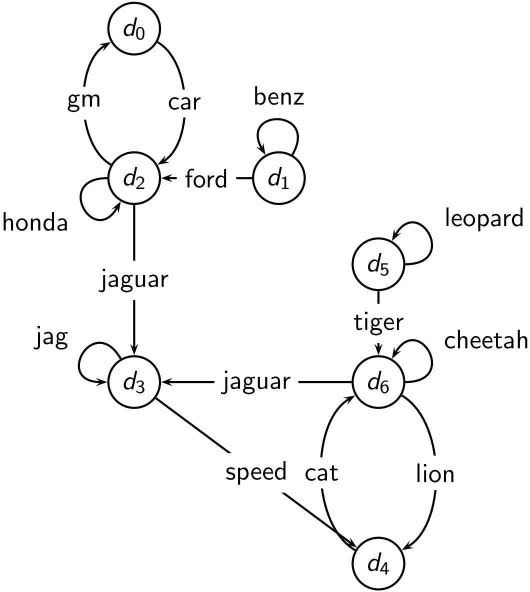
PageRank summary

- Preprocessing
  - Given graph of links, build matrix P
  - Apply teleportation
  - From modified matrix, compute $\vec{\pi}$
  - $\vec{\pi}_i$ is the PageRank of page i.
- Query processing
  - Retrieve pages satisfying the query
  - Rank them by their PageRank
  - Return reranked list to the user

PageRank issues

- Real surfers are not random surfers.
  - Examples of nonrandom surfing: back button, short vs. long paths, bookmarks, directories – and search!
  - $\rightarrow$ Markov model is not a good model of surfing.
  - But it's good enough as a model for our purposes.
- Simple PageRank ranking (as described on previous slide) produces bad results for many pages.
  - Consider the query [video service]
  - The Yahoo home page (i) has a very high PageRank and (ii) contains both *video* and *service*.
  - If we rank all Boolean hits according to PageRank, then the Yahoo home page would be top-ranked.
  - Clearly not desirable
- In practice: rank according to weighted combination of raw text match, anchor text match, PageRank & other factors
- $\rightarrow$ see lecture on Learning to Rank

Example web graph



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PageRank(d2)< PageRank(d6): why?

	a	h
d_0	0.10	0.03
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d_3	0.47	0.18

Transition (probability) matrix

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.00	0.00	1.00	0.00	0.00	0.00	0.00
d_1	0.00	0.50	0.50	0.00	0.00	0.00	0.00
d_2	0.33	0.00	0.33	0.33	0.00	0.00	0.00
d_3	0.00	0.00	0.00	0.50	0.50	0.00	0.00
d_4	0.00	0.00	0.00	0.00	0.00	0.00	1.00
d_5	0.00	0.00	0.00	0.00	0.00	0.50	0.50
d_6	0.00	0.00	0.00	0.33	0.33	0.00	0.33

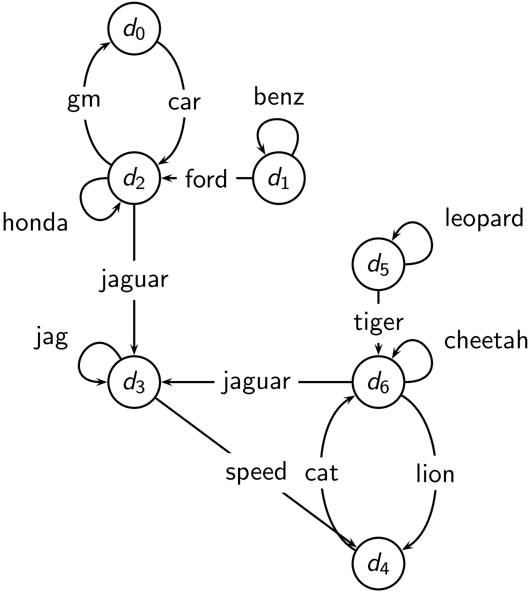
Transition matrix with teleporting

	d_0	d_1	d_2	d_3	d_4	d_5	d_6
d_0	0.02	0.02	0.88	0.02	0.02	0.02	0.02
d_1	0.02	0.45	0.45	0.02	0.02	0.02	0.02
d_2	0.31	0.02	0.31	0.31	0.02	0.02	0.02
d_3	0.02	0.02	0.02	0.45	0.45	0.02	0.02
d_4	0.02	0.02	0.02	0.02	0.02	0.02	0.88
d_5	0.02	0.02	0.02	0.02	0.02	0.45	0.45
d_6	0.02	0.02	0.02	0.31	0.31	0.02	0.31

Power method vectors $\vec{x}P^k$

	$\vec{x}$	$\vec{x}P^1$	$\vec{x}P^2$	$\vec{x}P^3$	$\vec{x}P^4$	$\vec{x}P^5$	$\vec{x}P^6$	$\vec{x}P^7$	$\vec{x}P^8$	$\vec{x}P^9$	$\vec{x}P^{10}$	$\vec{x}P^{11}$	$\vec{x}P^{12}$	$\vec{x}P^{13}$
d_0	0.14	0.06	0.09	0.07	0.07	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05
d_1	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
d_2	0.14	0.25	0.18	0.17	0.15	0.14	0.13	0.12	0.12	0.12	0.12	0.11	0.11	0.11
d_3	0.14	0.16	0.23	0.24	0.24	0.24	0.24	0.25	0.25	0.25	0.25	0.25	0.25	0.25
d_4	0.14	0.12	0.16	0.19	0.19	0.20	0.21	0.21	0.21	0.21	0.21	0.21	0.21	0.21
d_5	0.14	0.08	0.06	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
d_6	0.14	0.25	0.23	0.25	0.27	0.28	0.29	0.29	0.30	0.30	0.30	0.30	0.31	0.31

Example web graph



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d_4	0.16	0.04

How important is PageRank?

- Frequent claim: PageRank is the most important component of web ranking.
- The reality:
  - There are several components that are at least as important: e.g., anchor text, phrases, proximity, tiered indexes ...
  - Rumor has it that PageRank in its original form (as presented here) now has a negligible impact on ranking!
  - However, variants of a page's PageRank are still an essential part of ranking.
  - Adressing link spam is difficult and crucial. □